The Error Term

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Returns of tax-managed strategies will vary from the returns of target benchmarks. These differences are normal for managed portfolios, and especially worth accepting when they allow superior tax efficiency.

INVESTMENT PLANNING IS ABOUT STRUCTURING EXPOSURE TO RISK FACTORS. In any period random variation can make it hard to focus on the factors that drive returns. It is therefore important to expect statistical "noise" along the way and not let it unduly influence policy. The Fama-French multifactor model helps distinguish the systematic factors in returns from the random noise (Exhibit 1).

Exhibit 1

Three-Factor Model

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$$

Portfolio Excess Return | alpha | beta | size | BtM | error term

Priced Variation	Unpriced Variation
• Positive expected return	• Zero expected return
• Systematic	• Noise
• Economic	• Random
• Long-Term	• Short-Term
• Investing	• Speculating

This article owes a huge debt to discussions with Dave Butler and especially Eduardo Repetto, who provided data and inspiration.

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The three priced risk factors in equity returns are market, size, and book-to-market (BtM). All are "priced" because markets compensate investors with expected returns for taking them. The error term in the model (e(t)) captures all the residual, non-priced risk. In a well-diversified portfolio this can include variance from country or industry weights, differences in holdings, and even volatility from individual stocks. Such risks have no expected return.

Factor exposure determines expected return, but there are infinite ways to achieve any given factor exposure. For example, suppose you want mid-cap exposure. You can get it by holding mid-cap stocks. You can also get it by holding no mid-cap stocks at all but instead holding a combination of tiny stocks and huge stocks. Both portfolios might have identical factor exposures and identical expected returns, but inter-period returns are likely to differ dramatically.

Such differences are the result of residual error. They do not increase or decrease returns because the variance has no "direction"—differences from random holdings tend to average to zero through time. But since residual error is risk nonetheless, it is worth minimizing by building portfolios with similar composition to the target universe. Even then, portfolios with identical factor exposures will behave differently, as long as there are differences in their underlying securities.

Exhibit 2 shows three portfolios with the same factor exposures. For simplicity's sake the target is the Total Stock Market published by CRSP. The example portfolios have virtually identical, market-like factor exposures: around 1.00 on beta, 0.00 on size, and 0.00 on BtM.

Exhibit 2

Portfolio Combinations
January 1990–October 2001

	DFA U.S. Large Company Portfolio	Russell 1000 Index	CRSP 1-10 Index	CRSP 9-10 Index	Russell 2000 Index	Russell 3000 Index	Russell 3000 Growth Index
Market			100%				
Portfolio 1						100%	
Portfolio 2		90%			10%		
Portfolio 3	75%			10%			15%

	Size Co-efficient (s)	BtM Co-efficient (h)
Market	0.00	0.00
Portfolio 1	-0.04	0.04
Portfolio 2	-0.02	0.03
Portfolio 3	0.00	0.04

Exhibit 2 (continued)

	Market	Portfolio 1	Portfolio 2	Portfolio 3
Monthly				
Average Return	1.04	1.03	1.03	1.04
Standard Deviation	4.24	4.23	4.24	4.30
Tracking Error to Market	0.00	0.31	0.36	0.54
Maximum Over	0.00	1.55	1.68	2.99
Maximum Under	0.00	-1.28	-1.57	-1.46
Annualized				
Average Return	12.54	12.41	12.41	12.53
Standard Deviation	14.69	14.67	14.70	14.90
Tracking Error to Market	0.00	1.07	1.23	1.88
Rolling 6-Month Cumulative Return Difference (Rolling 6-Month Cumulative Return Tracking Error)				
Maximum Over	0.00	2.62	2.67	3.06
Maximum Under	0.00	-1.84	-2.09	-3.72

Portfolio 1 is the Russell 3000. This index is so much like the market that most of the institutional world uses it for a market benchmark. Portfolio 2 is a blend of large cap stocks (Russell 1000) and small cap stocks (Russell 2000). Portfolio 3 is a blend of S&P 500 and micro-cap stocks (CRSP 9-10), adding a large cap growth index (Russell 3000 Growth).

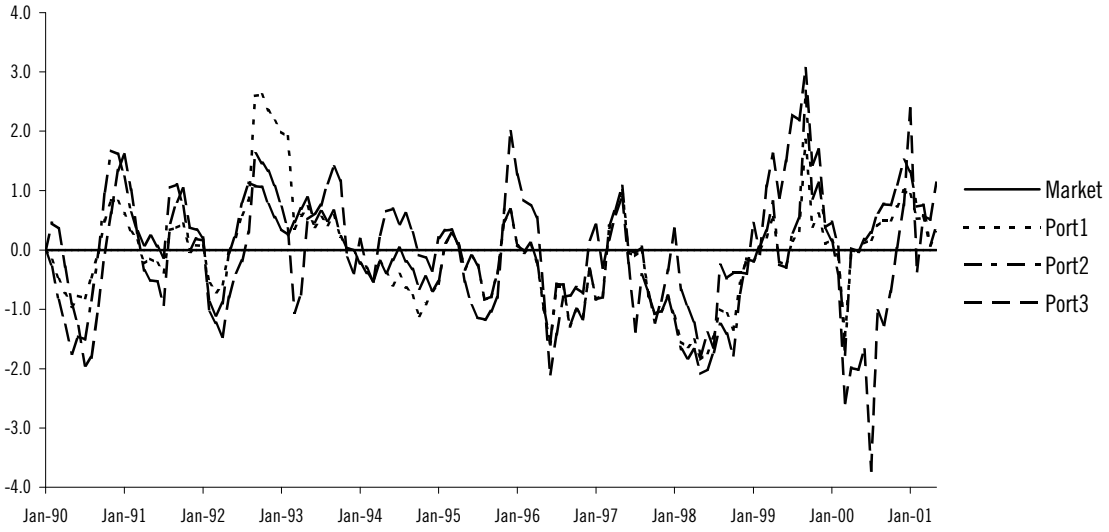
Since all of these portfolios have similar factors, they have similar expected returns. From January 1990 to October 2001, they also happen to have virtually identical realized returns (this will not always happen). Standard deviations of all four portfolios are similar, as are tracking differences (volatility of the premium) versus the market. All three portfolios are valid ways to capture diversified market-like exposure.

Yet residual variance still affects the returns. The maximum over- and under-performance versus the market in both monthly and cumulative six-month periods is significant for all three portfolios. Even the Russell 3000 Index, a portfolio we would expect to track the market, has a month where it under-performed by 128 basis points and an entire six-month span where it under-performed by 184 basis points. In spite of their varied structures, the other portfolios have similar highs and lows. The dispersion of securities among priced factors is not what causes these periodic differences—they result from residual error unrelated to systematic factors. This error is random: sometimes it's positive and sometimes negative. The periods of over-performance and under-performance tend to cancel each other out through time.

Exhibit 3 shows how wide the six-month cumulative difference due to residual error seems in plotted form. Tracking error like this can be distracting, but since the expected differences average to zero, managing this error should not take priority over managing the paying factors in returns. For many investors, foremost among these factors is taxes.

Exhibit 3

Tracking Error of Rolling Six-Month Cumulative Return
(Portfolio n - Market)
January 1990–October 2001



Most individual investors should consider taxes. After all, the expected return that comes from factor exposure has a wide variance around it. You will not get the average annual return every year. You will, however, be "asked" to pay taxes every year. The expected impact of taxes might be the most reliable explanatory factor in returns—the "known quantity."

The latest advance in multifactor engineering takes taxes into account. Dimensional has developed an algorithm that builds portfolios with targeted exposure to systematic factors like size and BtM, while "optimizing" the underlying set of securities to harvest capital losses and minimize dividends. The tax-managed versions of strategies can have specific factor exposures that are identical to non-tax managed alternatives or any other investor preference. But, as in the examples discussed above, the returns of the tax-managed portfolios and non-tax-managed alternatives will differ through time simply because the underlying securities differ. As in the earlier examples, these differences are random.

The only reason to make an example of tax-managed strategies is that tax management is such a worthwhile reason to accept random error. Differences between holdings in a tax-managed portfolio and its target universe exist because they lessen the tax burden. The long-term disadvantage of the error this causes is uncertain, but the strong advantages of tax-conscious investing are as certain as taxes themselves. In other words, investors should accept "noise" around the returns of some benchmark (which in the end is another arbitrary portfolio) in exchange for stronger expected returns after taxes.

Exhibit 4 shows two portfolios that, like those in the earlier example, target the market. As before, ten-year returns are shown, but this time simulating the effect of tax optimization. Case 1 applies moderate dividend management and Case 2 applies stronger dividend management.

Exhibit 4

Dividend Management

	CRSP Total Market Index	Dividend Management Case 1	Dividend Management Case 2
Annualized Average Monthly Return	13.94%	14.57%	15.11%
Annualized Average Dividend Yield	2.14%	1.62%	0.86%
Annualized Standard Deviation of the Monthly Returns	14.54%	14.94%	16.31%
Annualized Standard Deviation Tracking Error to Market		1.56%	3.41%
Correlation with Market	100%	99.47%	98.21%
Maximum Monthly Over-performance		2.16%	2.66%
Maximum Monthly Under-performance		-0.94%	-2.93%
Pre-Tax Growth of $1	3.09	3.35	3.57
After-Tax Growth of $1	2.75	2.98	3.26
Pre-Tax Annualized Compound Return	13.7%	14.3%	14.7%
After-Tax Annualized Compound Return	12.8%*	13.4%	14.1%

* No capital gains.

	CRSP Total Market Index		Dividend Management Case 1				Dividend Management Case 2			
	Return	Dividend Yield	Return	Dividend Yield	Return Difference	Yield Difference	Return	Dividend Yield	Return Difference	Yield Difference
7/90-12/90	-7.74%	1.68%	-8.58%	1.27%	-0.85%	-0.41%	-12.65%	0.72%	-4.91%	-0.96%
1991	33.59%	3.64%	35.85%	2.78%	2.26%	-0.87%	39.70%	1.64%	6.11%	-2.00%
1992	9.04%	2.79%	10.00%	2.13%	0.96%	-0.66%	11.68%	1.15%	2.64%	-1.64%
1993	11.50%	2.70%	10.23%	2.06%	-1.28%	-0.64%	11.34%	1.12%	-0.16%	-1.58%
1994	-0.60%	2.56%	0.89%	1.86%	1.50%	-0.70%	0.79%	0.98%	1.40%	-1.59%
1995	35.71%	2.83%	35.53%	2.12%	-0.18%	-0.71%	33.90%	1.04%	-1.81%	-1.80%
1996	21.27%	2.23%	22.74%	1.73%	1.47%	-0.50%	23.51%	0.94%	2.24%	-1.30%
1997	30.42%	1.96%	31.43%	1.53%	1.01%	-0.43%	30.52%	0.80%	0.10%	-1.16%
1998	22.55%	1.59%	24.85%	1.24%	2.30%	-0.35%	24.52%	0.65%	1.96%	-0.94%
1999	25.12%	1.41%	25.01%	1.02%	-0.10%	-0.39%	30.04%	0.52%	4.92%	-0.89%
2000	-11.04%	1.08%	-10.49%	0.79%	0.54%	-0.29%	-9.01%	0.39%	2.03%	-0.69%
1/01-6/01	-6.16%	0.48%	-6.13%	0.35%	0.03%	-0.13%	-7.28%	0.17%	-1.12%	-0.30%
Maximum Annual Over-performance				2.30%				6.11%		
Maximum Annual Under-performance				-1.28%				-4.91%		

Managing dividends, especially in an aggressive fashion, causes differences in underlying composition that affect tracking versus target portfolios. In this example, the moderate Case 1 had a maximum annual over-performance of 230 basis points and a maximum annual under-performance of 128 basis points versus the market. The aggressive Case 2 had a maximum annual over-performance of 611 basis points and a maximum annual under-performance of 491 basis points versus the market. Over- and under-performance in all cases is within the bounds of what you'd expect randomly.

Both cases have the same factor exposures and the same expected returns as the market. Cases 1 and 2, however, have significantly higher after-tax expected returns, especially in periods where dividend yields are high. In 1991, for instance, simulated Case 1 would have saved 0.87% in taxable dividends and simulated Case 2 would have saved 2.00% in taxable dividends. The contribution from tax management is expected to be positive regardless of the direction or magnitude of the investment return. The resulting increase in after-tax compound return is simulated in Exhibit 4.

When investing in tax-efficient strategies, investors make an implicit trade-off between tracking benchmarks and managing dividends. Structuring a diversified portfolio according to expectations and preference requires us to acknowledge and try to understand these trade-offs. This is easier when we recognize that benchmarks and published indexes are fundamentally arbitrary. They experience random noise in their returns just like managed portfolios do. Many investors will want to tolerate this noise in exchange for state of the art portfolio engineering and tax structure.

As always, the multifactor model helps us frame the problem. It helps us target factor exposures rather than benchmarks and helps us distinguish systematic expected returns from random noise. Most of all, it gives us the tools to build better portfolios and the perspective to stay disciplined during times when performance differs from the long-term expectation.