

Random Drift and Asset Allocation

DAVID G. BOOTH

FAMA AND FRENCH (1992) FIND THAT THREE FACTORS explain most of the differences in average stock returns,

- (1) the market factor, the stock market return in excess of the return on riskless assets,
- (2) the premium return of small cap stocks over large cap stocks, and
- (3) the premium return of value stocks over growth stocks.

The market factor explains little of the differences in average portfolio returns, because portfolio betas estimated from the three-factor model tend to cluster around 1.0. The size and value factors explain most of the differences in average portfolio returns.

The three factors appear to be random walks, with standard deviations that are large relative to the average values. As a result, there can be long periods of time when the factor returns drift one way or the other.

The purpose of this paper is to examine the factor drift in historical returns. The conclusion is that the lengths of the “cycles” for the factors are about what is expected by chance. The patterns that seem so obvious in the historical data are not predictable. They represent the normal drift in results that are characteristic of random walks.

A random walk is a desirable outcome. A random walk is the result of market prices incorporating all available information. If markets were chaotic, rather than accurately assessing risks, then the three-factor returns could very likely be predictable.

Dimensional Funds are offered by prospectus only, which contains more information about investment policies, charges, expenses, risks, and other matters of interest to the prospective investor. The material in this publication is provided solely as background information for registered investment advisors, institutional investors, and other sophisticated investors, and is not intended for public use. It should not be distributed to investors of products managed by Dimensional Fund Advisors Inc. or to potential investors. © 1999 by Dimensional Fund Advisors Inc. All rights reserved. Unauthorized copying, reproducing, duplicating, or transmitting of this material is prohibited.

Exhibit 1 displays the cumulative monthly returns for the three Fama/French (1992) risk factors since 1963. “ $R_M - R_F$ ” is the risk premium for stocks, the difference in returns between the CRSP Universe and one-month Treasury bills. “SmB” is the size effect, the difference in returns between small cap stocks and large cap stocks. “HmL” is the value effect, the difference in returns between high book-to-market and low book-to-market stocks.

There appear to be patterns in each of the three risk factors. Dimensional began investing in small cap stocks at the end of 1981. Since 1982, SmB has trended downwards. The seventeen-year compound return is 5.4% per year greater for the S&P 500 than for the Russell 2000 Index.

Exhibit 2 displays the comparison of large cap and small cap compound returns for two different periods in time. The CRSP 9-10 Index, an index of small cap stocks created by the Center for Research in Securities Prices at the University of Chicago, has a compound return from 1982-1998 that is almost identical to its compound return from 1926-1981. The compound return for the S&P 500 is about twice as great from 1982-1998 as it is from 1926-1981. Clearly, the negative size effect over the last 17 years is due to the unusually good performance of the S&P 500 rather than the poor performance of small cap stocks.

The cost of capital is the flip side of investment return. A company’s cost of capital is an investor’s investment return. The cost of equity capital for the largest, safest companies in the U.S., those in the S&P 500 Index, was 18.4% a year from 1982-1998, while the cost to smaller, more speculative companies was 11.3%. The implication is that the unusually good returns for the S&P 500 Index were unexpected by the companies themselves. It makes little sense that a low-risk company has to offer investors an 18.4% return in order to sell stock while a high-risk company has to offer an 11.3% return.

Given the ratio of average premium to standard deviation for the three risk factors, it is not unusual to find a seventeen-year cycle in performance. For the seventeen-year period immediately prior to the most recent one, 1965-1981, the compound return is lower for the S&P 500 than for Treasury bills. After that period, many investment committees questioned the commitment to equities. *Business Week* wrote an article entitled “The Death of Equities.” Those with weak beliefs in the relation between risk and expected return got out of equities and missed out on the best seventeen-year period for the S&P 500.

The historical data are filled with long runs of positive and negative returns for each of the three factors. Exhibits 3 through 5 divide historical returns into periods of up and down “cycles.” A 15% factor was used for the market factor, meaning that a new up or down market was established if the cumulative risk pre-

mium changed direction by more than fifteen percentage points. A 10% factor was used for SmB and HmL, reflecting their lower standard deviations.

The issue is whether or not the average length of a cycle in the historical data is about what we would expect from a random walk. Bootstrapping is a statistical technique that allows us to measure the expected average length of a cycle. A bootstrapping simulation was performed on the market factor, using 1,000 simulations of the last thirty-five years. For each simulated month, a return was drawn randomly from the thirty-five years of returns and used as the return for the month. The process was repeated each month for thirty-five years. At the end of each simulated thirty-five-year period, up and down markets were classified using the same criteria as in Exhibit 3.

Similar procedures were developed for SmB and HmL. Exhibit 6 displays the summary statistics for the three bootstrapping simulations, each simulating 1,000 thirty-five-year histories.

The average length of a cycle from 1965-1998 is about what is expected, based on the simulations, for each of the three risk factors. If anything, the average length of a cycle in historical returns is too short, rather than too long. The average cycle length for the equity risk premium ranks in the forty-eighth percentile of the distribution. A “perfect” score would be the fiftieth percentile. For none of the risk factors is the historical value in the lowest or highest ten percent of the estimated probability distribution.

Thus, the interpretation of Exhibit 1 is that it is a display of three random walks. The drift in returns for a factor is not predictable from its previous pattern of returns. Given the relation between average returns and standard deviations, all three series can drift in either direction for considerable periods of time.

The likelihood of long-term drift means that investors should have long-term horizons to invest in the overall stock market, and to emphasize small cap and value sectors. Many investors make decisions based on five or ten years of data. Given the drift in returns, a five-year or a ten-year period is largely statistical noise and is not long enough to determine if the expected factor returns have changed.

The unusual performance of large cap stocks is particularly strong the last four years. Exhibit 7 displays the four-year return for Decile 1 (largest) and Decile 10 (smallest) stocks, for 1995-1998. Deciles are formed based on NYSE rankings, and include AMEX and NASDAQ stocks as well. The four-year returns are ranked relative to the 829 possible four-year periods from January 1926 through December 1998. The 1995-1998 return of 33.1% for Decile 1 is the largest return since the four-year period ending March 1937. It ranks fifth out of all 829 possibilities,

and first out of all 600 possibilities over the last fifty years. Decile 10 stocks have a return that is only slightly above median. Measured as the difference between Decile 10 and Decile 1 compound returns, the size effect of -20.0% is the lowest in the last fifty years, ranking 822nd overall.

The extraordinary performance of large cap stocks over the last few years appears to be the result of a downward shift in the average cost of capital for large cap, but not small cap, stocks. Based on the dividend discount model, the price for a stock is

$$P = \frac{E}{r - g}$$

where r is the cost of capital and g is the growth rate of earnings. Exhibit 8 displays the changes in valuations for the S&P 500 Index and small cap stocks over the last five years. Displayed are the price-earnings ratios. The price-earnings ratio of the S&P 500 has nearly doubled, and, for the ten largest companies in the index, the ratio has more than doubled. Over the same period, the price-earnings ratio of small cap stocks has changed very little.

A doubling of the price-earnings ratio occurs when $r-g$ is cut in half. If $r-g$ is 6%, then the P/E ratio for a stock or index is 16.7. If $r-g$ is 3%, then the P/E ratio is 33.3.

Since current forecasts call for a modest growth in corporate earnings, the drop in $r-g$ for large cap stocks appears to be due to a reduction in the cost of capital rather than an increase in the growth rate of earnings. A sharp drop in the average cost of capital produces a large increase in stock prices.

In summary, all three of the Fama/French factor returns have long runs in performance. Over the last thirty-five years, the drift in factor returns is about what is to be expected from random walks.

The last four years has been a period of unusually good returns for large cap stocks, while being a normal period for small cap stocks. The strong relative performance is due in large part to a reduction in the average cost of capital for large cap stocks. Based on the current valuation ratios, small cap stocks have a "normal" expected return and large cap stocks have a "below normal" expected return. The expected premium return for small cap stocks is unusually high.

References

- Fama, Eugene F. and Kenneth R. French. "The Cross-Section of Expected Stock Returns." *Journal of Finance* 47 (1992).
- Fama, Eugene F. and Kenneth R. French. "Size and Book-to-Market Factors in Earnings and Returns." *Journal of Finance* 50 (1995).

Exhibit 1

Fama-French Three-Factor Cumulative Returns
Monthly Returns (%): July 1963-December 1998

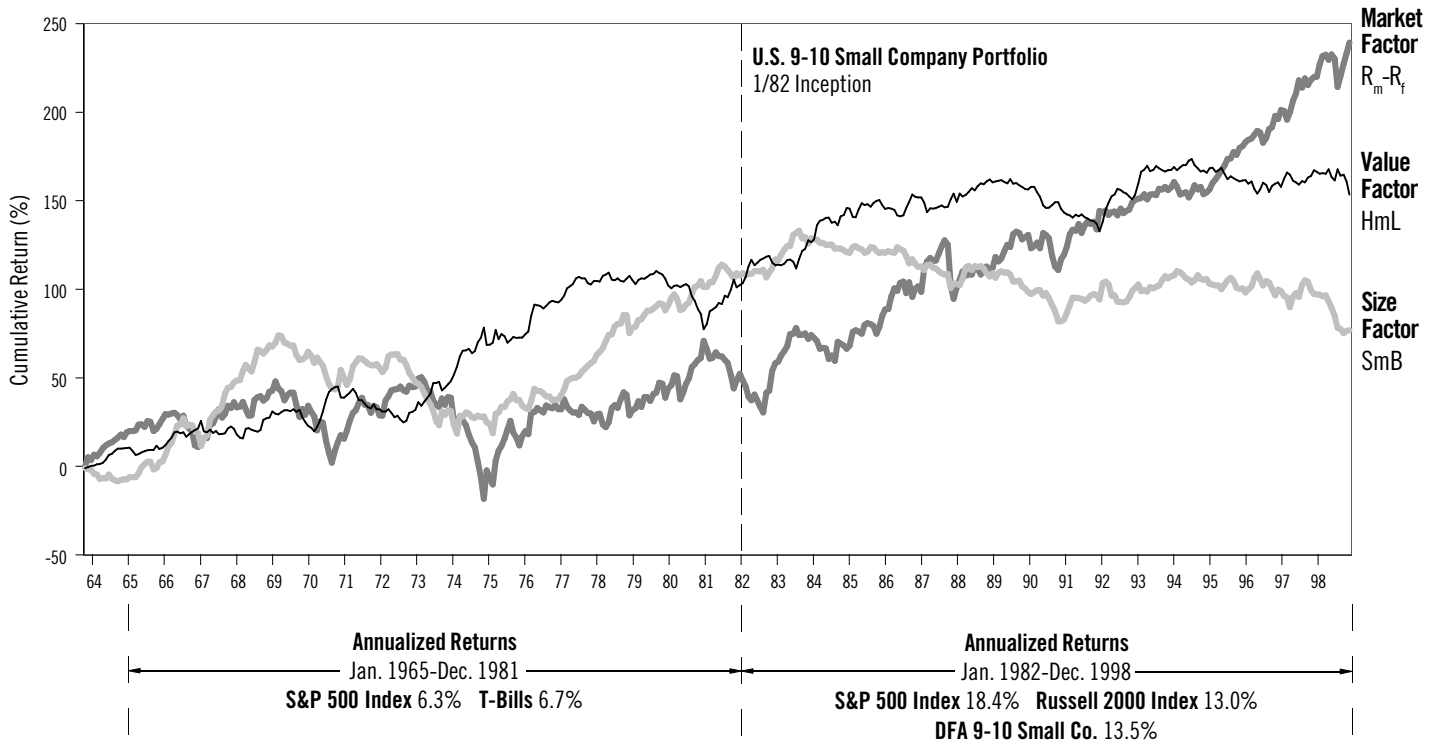


Exhibit 2

The Size Effect in Two Different Periods
1926-1981 versus 1982-1998

	1926-1981	1982-1998
CRSP 9-10 Index	11.8	11.3
S&P 500 Index	9.1	18.4
"Size Effect" (CRSP 9-10 minus S&P 500)	2.7	-7.1

- Small cap stocks match their historical average in the last 17 years.
- From 1982-1998, Dimensional's U.S. 9-10 Fund has an annualized return of 13.5%, outperforming the CRSP 9-10 Index by 2.2%.
- Large cap stocks have doubled their historical average in the last 17 years.

Exhibit 3

Up and Down Markets
Markets minus T-Bills
July 1963-August 1998

Up Markets					Down Markets				
	Months	$R_m - R_f$	SmB	HmL		Months	$R_m - R_f$	SmB	HmL
Jul. 1963-Jan. 1966	31	0.97	0.58	0.62	Feb. 1966-Sep. 1966	8	-2.41	0.09	0.28
Oct. 1966-Nov. 1968	26	1.44	1.96	0.32	Dec. 1968-Jun. 1970	19	-2.41	-1.35	0.73
Jul. 1970-Dec. 1972	30	1.60	0.01	-0.31	Jan. 1973-Sep. 1974	21	-3.27	-0.79	2.11
Oct. 1974-Dec. 1976	27	2.08	0.56	0.66	Jan. 1977-Feb. 1978	14	-1.13	1.94	0.86
Mar. 1978-Nov. 1980	33	1.47	0.92	-0.94	Dec. 1980-Jul. 1982	20	-2.01	0.49	1.99
Aug. 1982-Jun. 1983	11	4.32	1.87	-0.50	Jul. 1983-Jul. 1984	13	-1.43	-0.69	2.02
Aug. 1984-Aug. 1987	37	1.85	-0.39	0.22	Sep. 1987-Nov. 1987	3	-11.04	-1.73	2.55
Dec. 1987-Aug. 1989	21	1.81	0.06	0.29	Sep. 1989-Oct. 1990	14	-1.55	-1.61	-0.78
Nov. 1990-Jun. 1998	92	1.32	0.08	0.15	Jul. 1998-Aug. 1998	2	-9.18	-5.65	2.26
Average	34	1.61	0.42	0.09	Average	13	-2.47	-0.46	1.23

Exhibit 4

Up and Down Markets
Small minus Big
July 1963-October 1998

Up Markets					Down Markets				
	Months	$R_m - R_f$	SmB	HmL		Months	$R_m - R_f$	SmB	HmL
Jul. 1964-Apr. 1966	22	0.55	1.62	0.42	Jul. 1963-Jun. 1964	12	1.34	-0.70	0.84
Nov. 1966-Dec. 1968	26	1.13	2.40	0.14	May 1966-Oct. 1966	6	-2.26	-2.61	1.07
Aug. 1970-Apr. 1972	21	1.64	0.95	-0.75	Jan. 1969-Jul. 1970	19	-1.83	-1.60	0.80
Jan. 1974-Mar. 1974	3	-1.17	3.96	3.08	May 1972-Dec. 1973	20	-0.84	-2.25	1.38
Jan. 1975-Aug. 1978	44	1.18	1.52	0.78	Apr. 1974-Dec. 1974	9	-3.70	-1.32	0.49
Nov. 1978-Jul. 1983	57	0.79	1.01	0.16	Sep. 1978-Oct. 1978	2	-6.42	-5.21	1.65
Nov. 1987-Apr. 1988	6	1.01	2.10	0.69	Aug. 1983-Oct. 1987	51	0.56	-0.63	0.66
Nov. 1990-Feb. 1992	16	2.10	1.38	-0.26	May. 1988-Oct. 1990	30	0.08	-1.04	-0.19
Sep. 1992-Feb. 1994	18	0.83	0.98	0.70	Mar. 1992-Aug. 1992	6	-0.24	-1.97	1.62
Feb. 1996-May 1996	4	1.56	2.73	-1.91	Mar. 1994-Jan. 1996	23	1.09	-0.53	-0.25
May 1997-Sep. 1997	5	3.83	3.02	-0.84	Jun. 1996-Apr. 1997	11	0.94	-1.74	0.97
Average	20	1.12	1.50	0.23	Oct. 1997-Oct. 1998	13	0.64	-2.29	0.31
					Average	17	-0.12	-1.27	0.56

Exhibit 5

Up and Down Markets
High minus Low
July 1963-August 1998

Up Markets					Down Markets				
	Months	$R_m - R_f$	SMB	HmL		Months	$R_m - R_f$	SMB	HmL
Jul. 1963-May. 1969	71	0.59	0.96	0.45	Jun. 1969-Dec. 1969	7	-1.90	-1.38	-1.77
Jan. 1970-Aug. 1970	8	-1.83	-1.62	3.16	Sep. 1970-Jun. 1972	22	1.32	0.67	-0.91
Jul. 1972-Jul. 1979	85	-0.02	0.35	1.00	Aug. 1979-Nov. 1980	16	1.83	0.68	-2.06
Dec. 1980-Jun. 1989	103	0.52	0.07	0.82	Jul. 1989-Dec. 1991	30	0.68	-0.47	-0.98
Jan. 1992-Jul. 1994	31	0.33	0.30	1.32	Aug. 1994-May. 1996	22	1.58	0.24	-0.89
Jun. 1996-Aug. 1998	27	0.91	-1.15	0.51	Average	19	1.03	0.07	-1.18
Average	54	0.35	0.22	0.87					

Exhibit 6

Three-Factor "Bootstrap" Simulation
1963-1998 Experience Compared to the Results of 1,000 Simulations

	Up Markets	Down Markets
Market Factor: $R_m - R_f$ ($\pm 15\%$)		
Average Length (Months)		
Simulation Average	31.2	12.7
1963-1998 Experience		
Average	34.2	12.7
Percentile Rank	61	48
Size Factor: SMB ($\pm 10\%$)		
Average Length (Months)		
Simulation Average	27.8	19.2
1963-1998 Experience		
Average	20.2	16.8
Percentile Rank	83	64
Value Factor: HmL ($\pm 10\%$)		
Average Length (Months)		
Simulation Average	46.8	17.0
1963-1998 Experience		
Average	54.2	19.4
Percentile Rank	26	27

Exhibit 7

48-Month Annualized Returns (%)
January 1926-December 1998

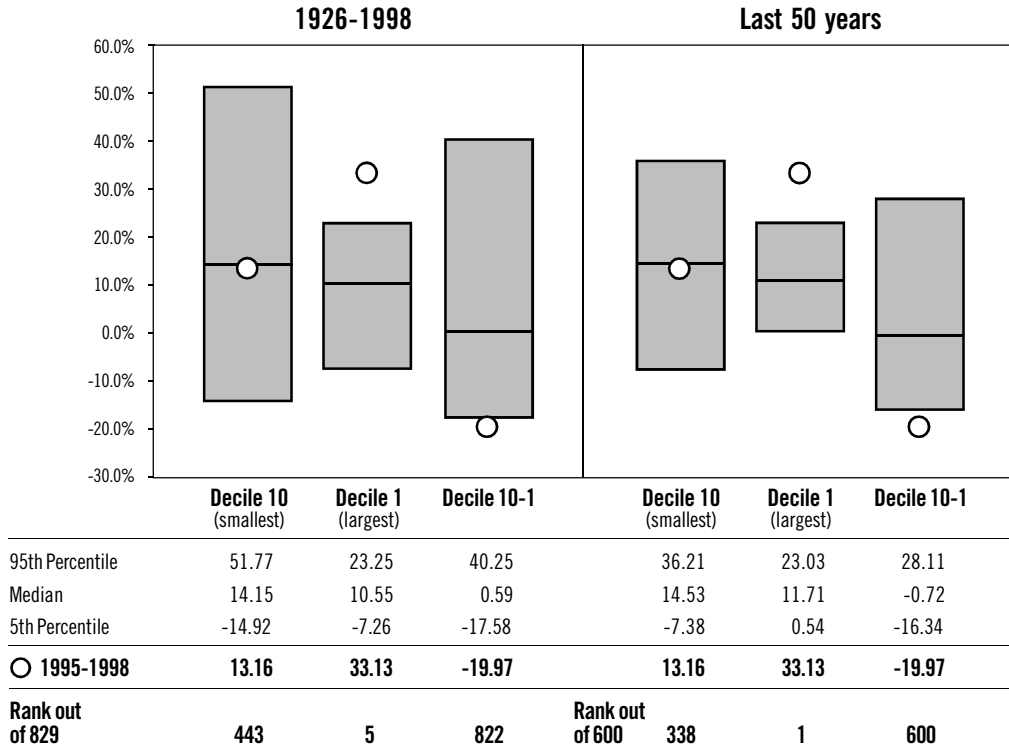
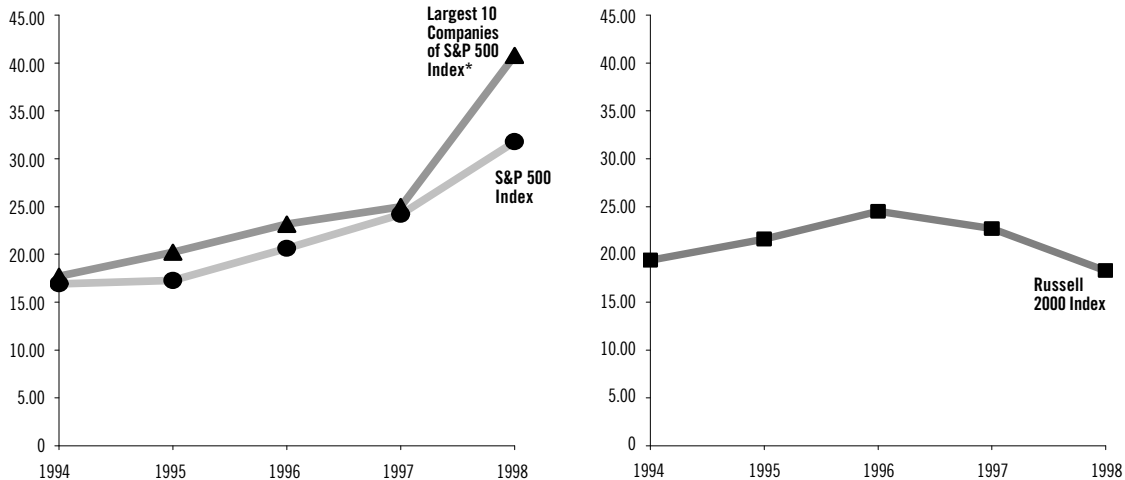


Exhibit 8

Price-Earnings Ratios
Year-end Ratios
1994-1998



*Company ratios are value-weighted and based on year-end market capitalization, subject to change annually.