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Update of the Research Underlying Dimensional's Bond Strategies

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Background

DIMENSIONAL'S BOND PRODUCTS ARE BASED ON RESEARCH I did in the 1980s. (See the bibliography at the end of the paper.) One of the results of this work was evidence that over short horizons (like a year) changes in interest rates (bond yields) are largely unpredictable. This result implies that current prices of discount bonds are good estimates of the prices of bonds with the same maturities one period from now. More interesting, expected returns on bonds can be inferred from current bond prices.

For example, if changes in interest rates are unpredictable, the expected price of a five-year discount bond held for one year (when it will be a four-year bond) is just the current price of a four-year bond. Thus, the expected gross return on the five-year bond—the expected growth in the price of the bond—is the ratio of the current prices of four-year and five-year discount bonds,

$$1 + E(R_5) = P_4/P_5. \quad (1)$$

In other words, when interest rate movements are unpredictable, the current price of a four-year bond is a good estimate of the price the current five-year bond will have one year from now, when it is a four-year bond.

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The ratio of four- and five-year bond prices in equation (1) also defines the five-year forward rate, that is, the forward interest rate for the year from four to five years from now,

$$(2) \qquad 1 + F(4:5) = P_4/P_5.$$

For the curious, the forward interest rate, F(4:5), is the one year interest rate for the year from four to five years from now that one can (in principle) lock in now. To see how this is done, suppose discount bonds pay $1 at maturity. To lock in the rate F(4:5), one shorts the four-year bond now, promising to pay $1 in four years in exchange for P_4 today. One then invests the proceeds, P_4, in P_4/P_5 units of the five-year bond. In four years one pays off the short sale with $1 out of pocket. In five years, one gets P_4/P_5 dollars from the five-year bond. One in effect commits now to invest $1 four years from now to get P_4/P_5 dollars for sure in five years.

The unpredictability of changes in interest rates has a simple implication that is the basis of Dimensional's bond strategies. Specifically, the term structure of current forward rates is the term structure of current one-year expected returns. To maximize expected return, one simply chooses the bond maturity that produces the largest forward rate.

The research behind this strategy is now about 20 years old. The strategy has worked well. Dimensional's bond funds consistently get high ratings from Morningstar, based on return to variability ratios. This suggests that my original empirical results hold up in the subsequent "out-of-sample" period. This paper updates the original research to test whether this is in fact the case.

Regression Tests

One implication of equation (1) that plays a big role in my original tests is that variation in forward rates should on average show up one-for-one as variation in returns one year ahead. This means that if one does a regression of, for example, the one-year return on a five year bond from t to t+1, $R_{5,t+1}$ on the five-year forward rate observed at t,

$$(3) \qquad R_{5,\, t+1} = a + bF_t(4:5) + e_{t+1},$$

the slope b on the forward rate should be indistinguishable from 1.0.

It turns out that regression (3) doesn't provide much of a test. The problem is that returns and forward rates tend to be consistently high or low for long periods. The regression picks this up, typically with a slope close to 1.0, even if there is not much short-term (for example, year-to-year) correspondence between variation in forward rates and variation in returns. To create a tougher test, I subtract the time t one-year interest rate, S_t (called the spot rate), from the return on the left hand side of (3) and from the forward rate on the right,

$$R_{5,\, t+1} - S_t = a + b[F_t(4{:}5) - S_t] + e_{t+1}. \quad (4)$$

Subtracting the spot rate from both sides of the regression (3) keeps it form just picking up variation in the general level of rates. As a result, regression (4) provides a stronger test of whether variation in the forward rate $F_t(4{:}5)$ shows up one-for-one as variation in the return $R_{5,\, t+1}$, in the short-term as well as in the long-term.

To present the updated empirical work, some term structure jargon is handy. $R_{5,\, t+1} - S_t$ is the one-year return on a five-year bond from t to t+1 in excess of the one-year interest rate. Such excess returns are called term premiums. $F_t(4{:}5) - S_t$ is the excess of the forward rate observed at time t, for the year four to five years ahead, over the one-year spot rate, also observed at t. These are called forward-spot spreads. And note that the one-year spot rate, S_t, is $R_{1,\, t+1}$, the return on a one-year bond from t to t+1. Unlike the one-year returns on longer-term bonds, the one-year return on a one-year bond is known when the bond is purchased; it is just the interest rate on the bond.

The data in the last of the original papers (Fama and Bliss (1986)) end in 1985. Thus, the update period in the new tests is 1986-2002. To put everything in perspective, results are shown for the full sample period (1953-2002), the original test period (1953-1985), and the new out-of-sample period (1986-2002). Moreover, the five-year term premium in regression (4) is just an illustrative maturity. Table 1 summarizes separate estimates of the regression,

$$R_{m,\, t+1} - S_t = a + b[F_t(m\text{-}1{:}\ m) - S_t] + e_{t+1}, \quad (5)$$

for maturities m from two to five years. Again, the proposition is that if changes in interest rates are unpredictable, variation through time in forward rates matches the variation in the expected returns on bonds with corresponding maturities. This means that the slopes b in (5) should be indistinguishable from 1.0 for all maturities m.

This prediction does well in the regressions for the original 1953-85 period (Table 1). The slope b for five-year bonds, 0.66, seems rather far from 1.0. But the standard errors of the slopes are large, and 0.66 is actually within one standard error of 1.0. The slopes for two-year to four-year maturities are closer to 1.0.

The conclusions are much the same for the 1986-2002 update period. The slope for 5-year bonds, 0.92, is now much closer to 1.0. The slope for two-year bonds, 0.54, is further away, but again within one standard error of 1.0. And some bouncing around of the slopes (across maturities and across time periods) is to be expected by statistical chance.

Table 1

Regressions of Term Premiums on Forward-Spot Spreads

$$R_{m,\,t+1} - S_t = a + b[F_t(m\text{-}1{:}\ m) - S_t] + e_{t+1}$$

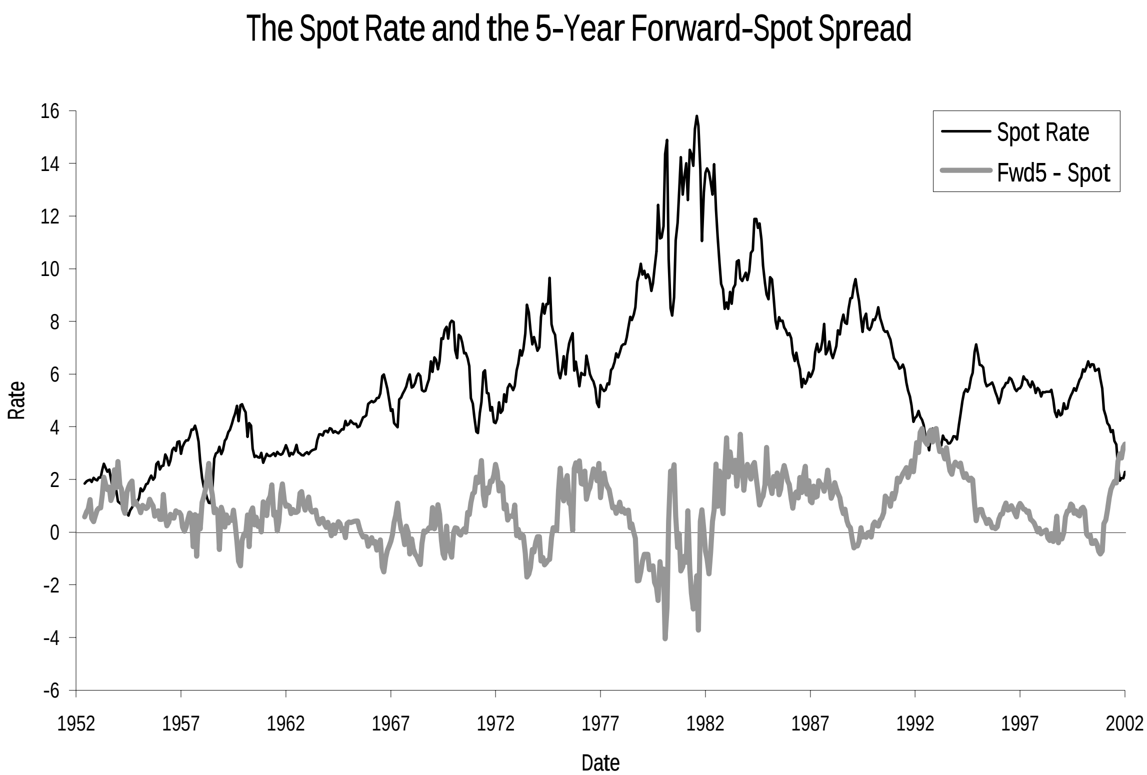
$R_{m,\,t+1}$ is the return for the year from t to t+1 on a bond with m years to maturity at t. S_t is the one-year spot rate observed at t. F_t(m1: m) is the forward rate observed at t for the year from t+m-1 to t+m. The three variables cover annual periods, but they are observed monthly. The standard errors of the regression coefficients, s(a) and s(b), are adjusted for autocorrelation due to the overlap of 11 monthly observations on the term premiums in annual returns with the method of Hansen and Hodrick (1980). The tstatistics t(a) and t(b) are the regression coefficients divided by their standard errors. The residual standard error, s(e), and the regression R^2 are adjusted for degrees of freedom.

	a	b	s(a)	s(b)	t(a)	t(b)	s(e)	R2
The period is 6/53-12/02, N=595 Months								
m = 2	0.06	0.86	0.26	0.29	0.22	3.02	1.66	0.11
m = 3	-0.09	1.11	0.48	0.37	-0.18	3.04	3.00	0.11
m = 4	-0.37	1.39	0.65	0.42	-0.57	3.32	4.10	0.13
m = 5	-0.13	1.03	0.84	0.51	-0.15	2.00	5.25	0.05
The period is 6/53-12/85, N=391 Months								
m = 2	-0.14	0.78	0.29	0.33	-0.48	2.35	1.72	0.09
m = 3	-0.40	0.96	0.52	0.42	-0.77	2.30	3.04	0.08
m = 4	-0.74	1.13	0.70	0.51	-1.05	2.21	4.11	0.07
m = 5	-0.76	0.66	0.92	0.58	-0.83	1.14	5.16	0.02
The period is 1/86-12/02, N=204 Months								
m = 2	0.68	0.54	0.50	0.51	1.36	1.07	1.44	0.04
m = 3	0.95	0.83	1.02	0.70	0.94	1.18	2.78	0.05
m = 4	0.91	1.16	1.35	0.72	0.67	1.60	3.91	0.10
m = 5	1.61	0.92	1.60	0.91	1.01	1.01	5.03	0.04

The best tests are obtained with the statistical power of the 1953-2002 half-century of available data. For the full period, the b slopes in regression (5) for two-year and five-year bonds, 0.86 and 1.03, are both much closer to 1.0 than in the two shorter periods, and b slopes for all maturities are within one standard error of 1.0. In short, the full period tests provide good evidence that variation through time in two-year to five-year forward rates is on average variation in the expected one-year returns on bonds of these maturities.

Other features of the results in Table 1 are worth noting. The regression R^2, which measure statistical fit (proportions of variance explained), are small, ranging from 0.05 to 0.13 in the regressions for the full sample period. This says there is lots of variation in realized returns which is not due to variation in expected returns. In other words, over one-year periods, bond returns have a big unexpected component, about which one can do nothing.

Figure 1



More interesting, the estimates of the intercept a are close to zero in the regressions for the overall 1953-2002 sample period. The intercepts for 1953-1985 are, however, systematically negative, while those for 1986-2002 are positive. The story behind these results is in Figure 1, which plots the spot rate (meant to be a proxy for the level of interest rates) and the five-year forward-spot spread $F_t(4{:}\ 5) - S_t$.

There is no particular trend in the forward-spot spread. It is typically (but not always) positive, and it wanders about with no particular long-term trend. (When the forward-spot spread is negative the term structure is predicting that one-year bonds have higher expected returns than five-year bonds.) In contrast, though the one-year spot rate also has lots of short-term up and down movement, its overall trend is generally up until mid-1981 (it peaks at 15.8% in August). Thereafter the spot rate drifts slowly down, but again with lots of short-term up and down movement. Not surprisingly, this long-term pattern in the spot rate corresponds to the long-term pattern in inflation during 1953-2002.

The regression intercepts in Table 1 measure the part of average annual bond returns left unexplained by forward rates. The intercepts suggest that the long upswing and the long downswing in the level of rates during the last half-century largely come as a surprise to the bond market. For 1953-1985 the intercepts are negative and they are more negative for longer maturity bonds. This is what one expects if the long upswing in rates during the period is a surprise. Surprise increases in interest rates are bad news for bond returns, and they hit long-maturity bond returns harder than short-maturity returns. Conversely, the intercepts for 1986-2002 are positive, and they are larger for long-maturity bonds. This suggests that the downward drift in interest rates during the period is largely a surprise. For the overall 1953-2002 period, however, the surprises wash out; the level of interest rates at the end of the period is about where it was at the beginning. And the regression intercepts in Table 1 are close to zero.

Average Term Premiums and Forward-Spot Spreads

There is another way to see the effect of unexpected changes in interest rates on bond returns. If changes in interest rates are unpredictable, the average term premium for a given maturity bond should be equal to the average value of the forward-spot spread for that maturity. Table 2 shows average term premiums and forward-spot spreads for the three time periods examined in Table 1.

For my original 1953-1985 period, average term premiums do not line up well with average forward-spot spreads. The average forward-spot spreads increase with maturity, from 0.23% (23 basis points per year) for two-year bonds to 0.56% (56 basis points) for five-year bonds. But for 1953-1985 average term premiums decrease with maturity, and the average premiums for four-year and five-year bonds are negative. For 1986-2002 average forward-spot spreads increase more strongly with maturity, from 0.67% for two-year bonds to 1.26% for five-year bonds. Average term premiums increase even more sharply, from 1.05% for two-year bonds to 2.77% for five-year bonds.

In short, during both 1953-1985 and 1986-2002 average forward-spot spreads increase with maturity. But during 1953-1985 average realized term premiums decrease with maturity, and during 1986-2002 average term premiums increase with maturity even more strongly than predicted by forward-spot spreads. What produces these contrasting results? My answer, long and short, is - chance. Unexpected changes in interest rates that are on balance positive during 1953-1985 depress the average returns on longer-term bonds, while unexpected declines in interest rates inflate the average returns on longer-term bonds during 1986-2002.

For the overall 1953-2002 sample period, unexpected positive and negative changes in interest rates largely even out, and average term premiums are quite close to average forward-spot spreads (Table 2).

What does all this say about Dimensional's bond products. To some extent the fine performance of the funds over the last 20 years is luck. The overall path of interest rates during the live period is down, and, as outlined above this creates unexpectedly high returns for longer term bonds. Since the term structure of forward

Table 2

Summary Statistics for Annual Forward-Spot Spreads and Term Premiums

The term premium for maturity m is $R_{m,\,t+1} - S_t$, where $R_{m,\,t+1}$ is the return for the year from t to t+1 on a bond with m years to maturity at t, and S_t is the one-year spot rate observed at t. The forward-spot spread for maturity m is $F_t(m\text{-}1{:}\ m) - S_t$, where $F_t(m1{:}\ m)$ is the forward rate observed at t for the year from t+m-1 to t+m. The variables cover annual periods, but they are observed monthly. Mean indicates an average value, Std is a standard deviation, and t(Mean) is the ratio of Mean to its standard error. Auto1 to Auto5 are autocorrelations of the term premium or the forward-spot spread at annual lags from 1 to 5 years.

	Mean	Std	t(Mean)	Auto1	Auto2	Auto3	Auto4	Auto5
Monthly Values of Annual Forward - Spot Spreads, 6/53-12/02, 595 Observations								
m = 2	0.38	0.69	13.51	0.41	0.12	-0.12	-0.09	0.05
m = 3	0.66	0.97	16.62	0.42	0.05	-0.18	-0.16	0.00
m = 4	0.81	1.15	17.09	0.47	0.13	-0.13	-0.15	-0.04
m = 5	0.80	1.22	16.04	0.37	0.11	-0.16	-0.15	-0.05
Monthly Values of Annual Term Premiums, 6/53-12/02, 595 Observations								
m = 2	0.39	1.76	5.35	0.16	-0.05	-0.02	-0.09	-0.05
m = 3	0.65	3.19	4.96	0.10	-0.07	0.02	-0.09	-0.09
m = 4	0.75	4.40	4.17	0.07	-0.05	0.06	-0.07	-0.12
m = 5	0.69	5.39	3.13	0.04	-0.06	0.06	-0.06	-0.13
Monthly Values of Annual Forward - Spot Spreads, 6/53-12/85, 391 Observations								
m = 2	0.23	0.70	6.63	0.36	-0.09	-0.37	-0.23	0.00
m = 3	0.45	0.97	9.17	0.35	-0.15	-0.38	-0.26	-0.02
m = 4	0.51	1.05	9.63	0.35	-0.13	-0.36	-0.28	-0.07
m = 5	0.56	1.19	9.30	0.25	-0.08	-0.33	-0.23	-0.08
Monthly Values of Annual Term Premiums, 6/53-12/85, 391 Observations								
m = 2	0.04	1.80	0.46	0.04	-0.06	-0.16	-0.27	-0.14
m = 3	0.03	3.18	0.17	-0.01	-0.07	-0.12	-0.27	-0.18
m = 4	-0.16	4.27	-0.76	-0.05	-0.06	-0.08	-0.26	-0.22
m = 5	-0.39	5.22	-1.49	-0.05	-0.08	-0.09	-0.25	-0.22
Monthly Values of Annual Forward - Spot Spreads for 1/86-12/02, 204 Observations								
m = 2	0.67	0.58	16.45	0.24	0.15	-0.19	-0.34	-0.06
m = 3	1.07	0.84	18.16	0.39	0.07	-0.37	-0.43	-0.17
m = 4	1.38	1.14	17.36	0.44	0.08	-0.32	-0.43	-0.22
m = 5	1.26	1.13	15.90	0.45	0.10	-0.32	-0.42	-0.24
Monthly Values of Annual Term Premiums for 1/86-12/02, 204 Observations								
m = 2	1.05	1.47	10.18	0.00	-0.26	-0.28	-0.30	0.11
m = 3	1.84	2.86	9.18	-0.10	-0.22	-0.22	-0.30	0.11
m = 4	2.51	4.11	8.71	-0.17	-0.18	-0.17	-0.30	0.12
m = 5	2.77	5.13	7.72	-0.26	-0.13	-0.11	-0.29	0.13

rates is mostly upward sloping during the period, Dimensional's bond portfolios were long most of the time, so they profited from the unexpected decline in rates. But as illustrated in Figure 1, there is also lots of wandering in forward-spot spreads during the period. The implication of regression slopes close to 1.0 in Table 1 is that the wanderings of forward-spot spread on average show up one-for-one as wanderings in term premiums. This is the basis of Dimensional's variable maturity strategies, and it is the source of their value added over fixed maturity strategies.

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